Potenze ad esponente intero.

BM3 Teoria pag. 30 – 32; esercizi pag. 91 – 93.

LM pag. 77; esercizi pag.106 – 110

Ricordi sicuramente che $2^4 : 2^3 = 2$; posso anche scrivere $\frac{2^4}{2^3} = \frac{2\cdot2\cdot2\cdot2}{2\cdot2\cdot2} = 2$

Applicando le stesse regole posso calcolare. $2^3 : 2^4 = 2^{-1}$;

che posso anche scrivere $\frac{2^3}{2^4} = \frac{2\cdot2\cdot2}{2\cdot2\cdot2\cdot2} = \frac{1}{2}$ dunque $2^{-1} = \left(\frac{2}{1}\right)^{-1} = \left(\frac{1}{2}\right)^{1} = \frac{1}{2} = 0{,}5$

In generale $a^{-n} = \left(\frac{a}{1}\right)^{-n} = \left(\frac{1}{a}\right)^{n} = \frac{1^n}{a^n} = \frac{1}{a^n}$; esempi: $(+3)^{-1} = \cdots\ldots\ldots\ldots$; $(-3)^{-1} = \cdots\ldots\ldots\ldots$;

$(+2)^{-2} = \cdots\ldots\ldots\ldots$; $(-2)^{-3} = \cdots\ldots\ldots\ldots$; $(-3x)^{-2} = \cdots\ldots\ldots\ldots$; $(+2m^2n^3)^{-2} = \cdots\ldots\ldots\ldots$;

Calcola:

$(+3)^{-2} = \cdots\ldots\ldots\ldots$; $(-3)^{-3} = \cdots\ldots\ldots\ldots$; $(-10)^{-2} = \cdots\ldots\ldots\ldots$; $(+9)^{-2} = \cdots\ldots\ldots\ldots$;

$(+5)^{-2} = \cdots\ldots\ldots\ldots$; $(-3)^{-3} = \cdots\ldots\ldots\ldots$; $(-5a^2)^{-2} = \cdots\ldots\ldots\ldots$; $(+4x^3y^5)^{-3} = \cdots\ldots\ldots\ldots$;

Con le frazioni avremo:

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n} = \frac{b^n}{a^n}$; esempi: $\left(+\frac{3}{2}\right)^{-1} = \cdots\ldots\ldots$; $\left(-\frac{3}{2}\right)^{-1} = \cdots\ldots\ldots\ldots$; $\left(-\frac{3}{2}\right)^{-3} = \cdots\ldots\ldots\ldots$;

Calcola: $\left(-\frac{5}{4}\right)^{-1} = \cdots\ldots\ldots\ldots$; $\left(-\frac{3}{2}\right)^{-2} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{5}{4}\right)^{-3} = \cdots\ldots\ldots = \cdots\ldots\ldots$;

$\left(+\frac{2}{3}\right)^{-3} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{1}{2}\right)^{-2} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{4}{3}\right)^{-4} = \cdots\ldots\ldots = \cdots\ldots\ldots$;

Con le lettere procedo allo stesso modo:

Calcola: $\left(-\frac{a}{b}\right)^{-1} = \cdots\ldots\ldots\ldots$; $\left(-\frac{x}{y}\right)^{-2} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{m}{n}\right)^{-3} = \cdots\ldots\ldots = \cdots\ldots\ldots$;

$\left(+\frac{2a}{3b}\right)^{-3} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{x}{2y}\right)^{-2} = \cdots\ldots\ldots = \cdots\ldots\ldots$; $\left(-\frac{4m}{3n}\right)^{-4} = \cdots\ldots\ldots = \cdots\ldots\ldots$;

Regola:

Con le potenze ad esponente intero valgono le ....note proprietà:

1) $(+2)^3 \cdot (+2)^4 = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots.. \left(+\frac{1}{2}\right)^3 \cdot \left(+\frac{1}{2}\right)^4 = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{1}{2}\right)^{-3} \cdot \left(+\frac{1}{2}\right)^4 = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots\ldots.; \left(+\frac{1}{2}\right)^{-3} \cdot \left(+\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

Esempi:

$\left(+\frac{3}{4}\right)^2 \cdot \left(+\frac{3}{4}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{1}{2}\right)^{-3} \cdot \left(-\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{5}{2}\right)^2 \cdot \left(+\frac{2}{5}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; (-2)^{-3} \cdot \left(-\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{2}{5}\right)^3 \cdot \left(-\frac{2}{5}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{2}{5}\right)^3 \cdot \left(+\frac{2}{5}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots$

$\left(+\frac{x}{y}\right)^2 \cdot \left(+\frac{x}{y}\right)^{-4} = \cdots \ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{2m}{3n}\right)^{-3} \cdot \left(-\frac{3n}{2m}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{x}{y}\right)^2 \cdot \left(+\frac{y}{x}\right)^{-4} \cdot \left(+\frac{x}{y}\right)^{-5} = \cdots \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..$

2) $(+2)^5 : (+2)^3 = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots.. \left(+\frac{1}{2}\right)^5 : \left(+\frac{1}{2}\right)^3 = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{1}{2}\right)^5 : \left(+\frac{1}{2}\right)^3 = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots\ldots.; \left(+\frac{1}{2}\right)^{-5} : \left(+\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

Esempi:

$\left(+\frac{3}{4}\right)^2 : \left(+\frac{3}{4}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{1}{2}\right)^{-3} : \left(-\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{5}{2}\right)^2 : \left(+\frac{2}{5}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; (-2)^{-3} : \left(-\frac{1}{2}\right)^{-4} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left(+\frac{2}{5}\right)^5 : \left(-\frac{2}{5}\right)^{-6} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{2}{5}\right)^5 : \left(+\frac{2}{5}\right)^{-6} = \cdots \ldots\ldots\ldots\ldots. = \cdots \ldots\ldots\ldots$

$\left(+\frac{x}{y}\right)^{12} : \left(+\frac{x}{y}\right)^{-6} = \cdots \ldots\ldots\ldots. = \cdots \ldots\ldots\ldots.; \left(-\frac{2m}{3n}\right)^{-12} : \left(-\frac{3n}{2m}\right)^{-5} = \cdots \ldots\ldots\ldots\ldots.. = \cdots \ldots\ldots\ldots\ldots..$

$\left[\left(+\frac{x}{y}\right)^{12} : \left(+\frac{x}{y}\right)^{-6}\right] \cdot \left(+\frac{y}{x}\right)^{18} =$ ....

....

3) $(+2^3)^4 = \cdots$ ……; $\left[\left(-\frac{1}{2}\right)^3\right]^4 = \cdots$ ……

$\left[\left(+\frac{1}{2}\right)^{-2}\right]^3 = \cdots$ ……; $\left[\left(-\frac{1}{2}\right)^{-2}\right]^{-3} = \cdots$ ……;

$\left[\left(-\frac{5}{3}\right)^3\right]^{-5} = \cdots$ ……; $[(+0{,}2)^{-6}]^2 = \cdots$ ……;

4) $(+2)^2 \cdot (+3)^2 = \cdots$ ……; $\left(+\frac{1}{2}\right)^3 \cdot \left(+\frac{4}{3}\right)^3 = \cdots$ ……

$\left(+\frac{5}{4}\right)^{-3} \cdot \left(+\frac{8}{9}\right)^{-3} = \cdots$ ……; $\left(-\frac{5}{4}\right)^{-3} \cdot \left(+\frac{8}{9}\right)^{-3} = \cdots$ ……

5) $(+12)^2 : (+3)^2 = \cdots$ ……; $\left(+\frac{20}{9}\right)^3 : \left(+\frac{4}{3}\right)^3 = \cdots$ ……

$\left(+\frac{10}{21}\right)^{-3} : \left(+\frac{8}{9}\right)^{-3} = \cdots$ ……; $\left(+\frac{10}{21}\right)^{-3} : \left(-\frac{8}{9}\right)^{-3} = \cdots$ ……

6) Potenze ad esponente intero e radicali. Calcola:

$(\sqrt{2})^{-2} = \cdots$ ……; $\left(\frac{1}{\sqrt{2}}\right)^{-2} = \cdots$ ……

$(\sqrt{a})^{-2} = \cdots$ ……; $\left(\frac{1}{\sqrt{x}}\right)^{-2} = \cdots$ ……

$(\sqrt{2})^{-2} \cdot (\sqrt{2})^{-3} = \cdots$ ……; $(\sqrt{m})^{-2} \cdot (\sqrt{m})^{-3} = \cdots$ ……

7) Espressioni con le potenze ad esponente intero.

Applico le ……note regole già viste con le potenze ad esponente naturale.

a) $\left(\frac{2^2}{2^4} - 2^{-2}\right)^8 + 2^4 =$ $\left[\frac{1}{16}\right]$

b) $\frac{7^{-1}-7^{-2}}{2^{-3}} \cdot \left(-\frac{7}{8}\right)^2 =$ $\left[\frac{3}{4}\right]$

c) $\left[6 \cdot \left(+\frac{3}{4}\right)^{-2}\right]^{-1} : \left[\left(+\frac{2}{9}\right)^{-2} : \left(+\frac{3}{2}\right)^4\right]^{-1} + 2^{-3} \cdot \frac{1}{3^{-3}} \cdot (+3)^{-2} =$ $\left[\frac{3}{4}\right]$

d) $\left[\left(-\frac{2}{5}\right)^2 \cdot \left(\frac{1}{4}\right)^2\right]^2 : (5^2 \cdot 2^2)^{-2} - \left(+\frac{4}{5}\right)^{-5} \cdot \left(+\frac{4}{5}\right)^{-4} : \left(+\frac{4}{5}\right)^{-7} =$ $\left[-\frac{9}{16}\right]$

e) $\left\{\left(+\frac{11}{3}\right)^{-2} : \left[\left(+\frac{6}{7}\right)^{-3} \cdot \left(+\frac{7}{11}\right)^{-2}\right]^{-1}\right\}^{-2} \cdot \left(-\frac{30}{35}\right)^{-3} =$ $\left[-\frac{56}{3}\right]$

f) $(+2a^3b) : (-5xy^2) =$

g) $\left(-\frac{3}{4}xy^3\right) : \left(-\frac{1}{2}x^4y^5\right) =$

h) $\left[\left(-\frac{a}{2}\right)^{-4}\right]^3 : \left[\left(-\frac{a}{2}\right)^2\right]^{-5} =$

i) $[(-x^2)^4]^{-3} \cdot [(-x)^{-3}]^{-2} : (x^{-2})^3 =$

j) $[(a^{-2})^{-1}]^{-1} + \left(-\frac{1}{2}a\right)^{-2} + \left[-\left(-\frac{1}{3}a^2\right)^{-1}\right] =$

Esercizi.

1) Scrivi sotto forma di frazione e calcola:

a) $(-3)^{-2} =$ **b)** $(+2)^{-5} =$ **c)** $(-4)^{-3} =$ **d)** $\left(+\frac{3}{10}\right)^{-3} =$

e) $\left(-\frac{5}{2}\right)^{-2} =$ **f)** $\frac{5}{2^{-3}} =$ **g)** $-\frac{8^{-1}\cdot 3^{2}}{2^{-2}} =$ **h)** $\left(\frac{5^{-1}}{2^{-2}}\right)^{-2} =$

2) Scrivi il numero con l'esponente positivo e il numero decimale:

a) $(+5)^{-1} =$ **b)** $(-5)^{-1} =$ **c)** $(-2)^{-3} =$ **d)** $(-10)^{-1} =$

e) $(+10)^{-2} =$ **f)** $(-2)^{-5} =$ **g)** $(+6)^{-3} =$ **h)** $(-3)^{-2} =$

3) Scrivi la frazione con **l'esponente positivo** e **calcola**.

a) $\left(+\frac{5}{2}\right)^{-1} =$ **b)** $\left(-\frac{5}{2}\right)^{-1} =$ **c)** $\left(+\frac{5}{2}\right)^{-2} =$ **d)** $\left(-\frac{5}{2}\right)^{-2} =$

e) $\left(-\frac{3}{7}\right)^{-1} =$ **f)** $\left(-\frac{3}{7}\right)^{-2} =$ **g)** $\frac{2^{3}}{2^{-5}} =$ **h)** $\frac{2^{-3}}{2^{-5}} =$

4) Scrivi la frazione **riducendo la base** al minimo fattore e calcola.

a) $(+4)^{-2} =$ **b)** $(-8)^{-2} =$ **c)** $(-27)^{-3} =$ **d)** $(-25)^{-2} =$

e) $\left(+\frac{4}{9}\right)^{-2} =$ **f)** $\left(-\frac{27}{8}\right)^{-2} =$ **g)** $\left(-\frac{125}{64}\right)^{-2} =$ **h)** $\left(+\frac{0{,}01}{0{,}09}\right)^{-2} =$

5) Con **le lettere** procedo allo stesso modo; scrivi la potenza con **l'esponente positivo**.

a) $(+\mathrm{a})^{-1} =$ **b)** $(-\mathrm{b})^{-1} =$ **c)** $(+2\mathrm{a})^{-1} =$ **d)** $(-10\mathrm{m})^{-1} =$

e) $(+10\mathrm{x}^{3})^{-2} =$ **f)** $(+10\mathrm{x}^{3}y^{2})^{-2} =$ **g)** $(+10\mathrm{x}^{3}y^{-2})^{-2} =$ **h)** $(+10\mathrm{x}^{-3}y^{-2})^{-2} =$

6) Con **le lettere** procedo allo stesso modo; scrivi la frazione con **l'esponente positivo**.

a) $\left(+\frac{\mathrm{a}}{b}\right)^{-1} =$ **b)** $\left(-\frac{\mathrm{b}}{a}\right)^{-1} =$ **c)** $\left(+\frac{\mathrm{x}}{y}\right)^{-2} =$ **d)** $\left(+\frac{\mathrm{q}^{-1}}{p^{2}}\right)^{2} =$

e) $\left(+\frac{\mathrm{q}^{-1}}{p^{-2}}\right)^{2} =$ **f)** $\left(+\frac{\mathrm{q}^{-1}}{b^{-2}}\right)^{-2} =$ **g)** $\left(+\frac{3\mathrm{m}^{2}}{2n^{4}}\right)^{-2} =$ **h)** $\left(+\frac{3^{-1}\mathrm{m}^{-2}}{2n^{-4}}\right)^{-2} =$

7) Scrivi il numero sotto forma di **frazione** con **l'esponente positivo**.

a) $(-2,\overline{5})^{-1} =$ **b)** $(-2,4\overline{5})^{-1} =$ **c)** $\left(-2\frac{1}{2}\right)^{-1} =$ **d)** $(-25\%)^{-1} =$

e) $\left(-\sqrt{2}\right)^{-1} =$ **f)** $\left(-\sqrt{2}\right)^{-1} =$ **g)** $\left(-\sqrt{8}\right)^{-2} =$ **h)** $\left(+\sqrt{4^{-2}}\right)^{-1} =$

8) Calcola applicando le.....note **proprietà delle potenze** e scrivendo il risultato con l'esponente positivo.

a) $\left(-\frac{2}{3}\right)^{-5}\cdot\left(-\frac{2}{3}\right)^{-4} =$ **b)** $\left(-\frac{2}{3}\right)^{-5}\cdot\left(+\frac{2}{3}\right)^{-4} =$ **c)** $\left(+\frac{2}{3}\right)^{-5}\cdot\left(+\frac{3}{2}\right)^{-4} =$

d) $\left(-\frac{3}{2}\right)^{-1}:\left(+\frac{2}{3}\right)^{-4} =$ **e)** $\left(+\frac{4}{9}\right)^{-2}\cdot\left(-\frac{2}{3}\right)^{-3} =$ **f)** $\left(-\frac{4}{9}\right)^{-2}:\left(-\frac{8}{27}\right)^{3} =$

9) **Ora sei in grado** di svolgere gli esercizi su LM da pag. 106 – 110; non tutti 😉 !