Serie 12 - Tecnica di calcolo. 22 marzo 2016

1) Calcola il valore delle seguenti espressioni numeriche in $\mathbb{R}$

419 $\left(\frac{3^{-1}-2^{-2}}{3^{-1}+2^{-2}}\right)^{-1}+\left(\frac{2^4\cdot 2^8}{2^{12}}-\frac{2\cdot 3^9}{3^{10}}\right)^{-1}$ [10]

420 $\left[\left(-\frac{2}{3}\right)^{-1}\left(+\frac{3}{2}\right)^{2}\left(-\frac{2}{3}\right)^{-3}\right]:\left(-\frac{2}{3}\right)^{-8}$ $\left[\frac{4}{9}\right]$

421 $\frac{\left(\frac{1}{2}\right)^3\cdot\left[\left(\frac{1}{2}\right)^5\right]^2}{\left[\left(\frac{1}{2}\right)^2\right]^7}-\left[\left(\frac{1}{3}\right)^{13}:\left(\frac{1}{3}\right)^{10}\right]^{-1}$ [−25]

2) Calcola il valore delle seguenti espressioni con i radicali.

533 $(\sqrt{18}+\sqrt{50}) : \sqrt{2}+(\sqrt{5}+1)^2+(\sqrt{5}-1)(\sqrt{5}+1)$ $[18+2\sqrt{5}]$

534 $(\sqrt{500}-\sqrt{125}) : \sqrt{5}+(\sqrt{5}-\sqrt{2})^2-(\sqrt{10}-1)(\sqrt{10}+1)$ $[3-2\sqrt{10}]$

535 $\left(\sqrt{a}-\sqrt{b}\right)^2+\left(\sqrt{a}-2\sqrt{b}\right)^2+\left(\sqrt{3a}-\sqrt{b}\right)\left(\sqrt{3a}+\sqrt{b}\right)-2\sqrt{b}(2\sqrt{b}-3\sqrt{a})$ $[5a]$

3) Risolvi le seguenti equazioni a coefficiente irrazionale, specificando l'insieme soluzione.

856 $(x-\sqrt{2})^2=(x+\sqrt{2})(x+3\sqrt{2})$ $\left[-\frac{\sqrt{2}}{3}\right]$

857 $(x+\sqrt{2})(x-\sqrt{3})-(x+\sqrt{3})(x-\sqrt{2})=-2$ $[\sqrt{2}+\sqrt{3}]$

858 $\frac{1}{2}(x-\sqrt{2})+\frac{x+\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{2}-1}{\sqrt{2}+1}$ $[7\sqrt{2}-10]$

859 $x\sqrt{2}+\frac{1}{\sqrt{2}-1}=\sqrt{2}+1$ [0]

860 $\frac{1}{x-\sqrt{5}}+\frac{1}{x+\sqrt{5}}=\frac{2}{x^2-5}$ [1]

4) Risolvi le seguenti disequazioni a coefficiente irrazionale.

693 $2x\sqrt{20}-10\geq 6x\sqrt{5}$ $[x\leq -\sqrt{5}]$

694 $(\sqrt{3}-2)x>2\sqrt{3}-4$ $[x<2]$

695 $x(1-\sqrt{2})\geq\sqrt{2}-2$ $[x\leq\sqrt{2}]$

696 $(x-\sqrt{3})^2<x^2$ $\left[x>\frac{\sqrt{3}}{2}\right]$

697 $\sqrt{2}(x-\sqrt{3})+\sqrt{3}(x-\sqrt{2})\leq 4\sqrt{6}$ $[x\leq 18\sqrt{2}-12\sqrt{3}]$

698 $(x-\sqrt{2})^2+(x+\sqrt{2})^2\geq 2(x-\sqrt{2})(x+\sqrt{2})$ $[\forall\, x\in\mathbb{R}]$

5) Semplifica le seguenti espressioni algebriche

388 $\left(\frac{1}{x^2+1}-\frac{1}{2x+4}\right)\cdot\left(\frac{25}{2x-6}-\frac{1}{2x+2}+x+4\right)$ $\left[-\frac{1}{2}\right]$

389 $\left(\frac{a^2+2a}{a^2+4a+4}+\frac{a}{a^2-2a}\right)\cdot\frac{a^2-4}{(a^2+2)^2-a^2}$ $\left[\frac{1}{a^2+a+2}\right]$

390 $\left(\frac{x}{2x+2}-\frac{x}{2x-2}-\frac{2}{1-x^2}\right):\left(\frac{1}{2x-2}-\frac{9}{2x+2}+1\right)$ $\left[\frac{1}{2-x}\right]$

6) Equazioni fratte.

40 $\frac{1}{x^2+4x-5}-\frac{1}{6x-6}=\frac{1}{x+5}$ [Impossibile]

41 $\frac{1}{x^2-4x+4}+\frac{x}{x^2+x-6}=\frac{1}{x+3}$ $\left[\frac{1}{3}\right]$

42 $\frac{1}{x+3}-\frac{1}{x-2}=-\frac{5}{x^2+x-6}$ $[\forall x\in\mathbf{R}-\{-3,2\}]$

43 $\frac{2}{x^2+5x-6}-\frac{1}{x^2-1}=\frac{1}{x^2-3x-4}$ $\left[\frac{22}{13}\right]$

7) Risolvi i seguenti sistemi d'equazioni.

239 $\begin{cases}\frac{x-1}{2}=\frac{3+y}{3}\\(x-1)^2=(x-2)^2+y\end{cases}$ $[(-3,-9)]$

240 $\begin{cases}\frac{1}{2}\left(x-\frac{1}{3}\right)-\frac{1}{3}\left(x-\frac{1}{4}\right)=\frac{1}{6}+y\\(x-1)(x+2)=x(x+1)-y\end{cases}$ $\left[\left(\frac{27}{2},2\right)\right]$

241 $\begin{cases}\frac{x-y}{8}+\frac{x-6}{4}=\frac{x-y}{6}+\frac{x-2}{3}\\\frac{1}{2}(x-4)+\frac{1}{3}(3x-y)=-9\end{cases}$ $\left[\left(-\frac{2}{3},18\right)\right]$

8) Equazioni di grado superiore.

163 $(2x+1)(x-2)-(x-1)(x+1)=9$ $[-2,5]$

164 $2x^2=(x+2)(x-3)+12$ $[-3,2]$

165 $(x-3)^2+(5-x)^2=2x-6$ $[4,5]$

166 $(x-3)(x+1)-(x-2)^2=(x+1)(x-3)$ $[2]$

167 $(x+1)^3-x^3=7$ $[-2,1]$